ME 141
Engineering Mechanics

Portion 2
Introduction to Statics

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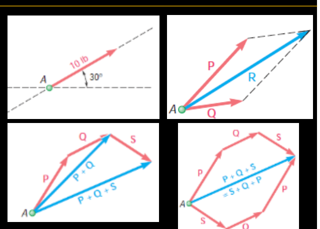
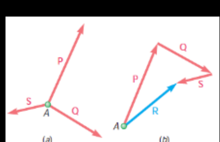
Statics of Particle

- Forces in a Plane (2D Analysis)
- Forces in a Space (3D Analysis)

Forces in a Plane
Forces on a Particle

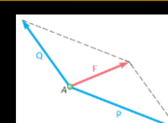
- Point Force
- Vector
- Resultant of Multiple Forces
- Addition of Vectors
- Triangle Rule
- Parallelogram Law
- Polygon Rule
- Resultant of Concurrent Forces

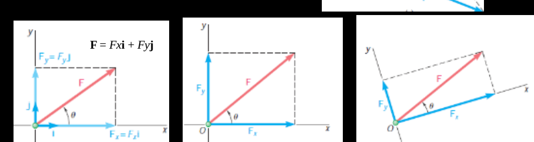




Forces in a Plane
Forces on a Particle

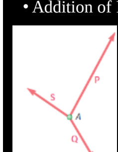
- Resolution of Forces into Components
- Rectangular Components
- Unit Vectors



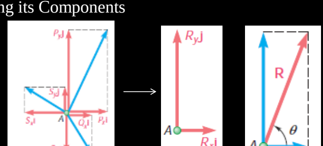


Forces in a Plane
Forces on a Particle

- Addition of Forces Using its Components



Resolving into Components



$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{S}$

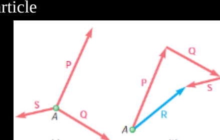
$$R_x\mathbf{i} + R_y\mathbf{j} = P_x\mathbf{i} + P_y\mathbf{j} + Q_x\mathbf{i} + Q_y\mathbf{j} + S_x\mathbf{i} + S_y\mathbf{j}$$
$$= (P_x + Q_x + S_x)\mathbf{i} + (P_y + Q_y + S_y)\mathbf{j}$$

$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$$

Forces in a Plane
Forces on a Particle

- Equilibrium of a Particle



The three forces will be in equilibrium if and only if

$$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$$
$$\sum Fx = \sum Fy = \mathbf{0}$$

Remember Newton

__If the resultant force acting on a particle is zero,__ the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

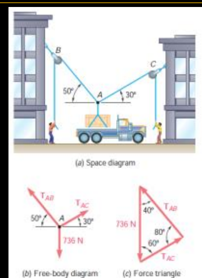
Forces in a Plane

Forces on a Particle

- Free Body Diagram of a Particle

Suppose 75 kg crate equivalent to 736 N weight is to be lifted using rope-pulleys.
We need to know whether the ropes can carry the load or not i.e. we need to determine the tension in the individual rope.

Steps:
- Draw a Free Body Diagram of the most significant point, here it is P.
- Resolve the components of the force or **use triangle rule** to find out two equations for the two unknown force.



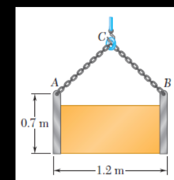
$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{AC}}{\sin 40^\circ} = \frac{736\ \text{N}}{\sin 80^\circ}$$

$$T_{AB} = 647\ \text{N} \qquad T_{AC} = 480\ \text{N}$$

Free Body Diagram is specially necessary in problem where a particle is in equilibrium condition.

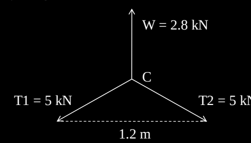
Problem 2.2 (Beer Johnston_10th edition_P2.62)

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling *ACB that can be used to lift* the loaded bin if the tension in the chain is not to exceed 5 kN.



Solution:

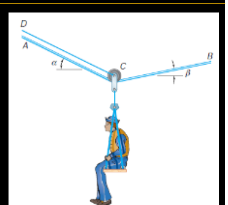
Free Body Diagram of Point C:



Ans.: *ACB* = 1.25 m

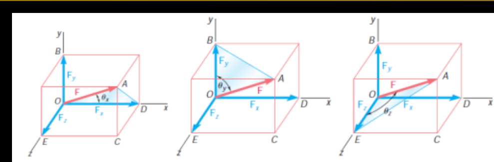
Problem 2.2 (Beer Johnston_10th edition_P2.53)

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB and* is pulled at a constant speed by cable *CD. Knowing that α = 30°, β= 10°* and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension *(a) in the support cable ACB, (b) in the traction cable CD.*



Forces in a Space

Forces on a Particle

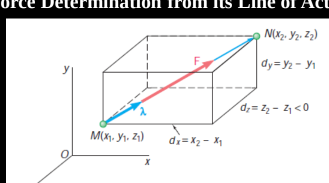


$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$
$$F_x = F\cos\theta_x$$
$$F_y = F\cos\theta_y$$
$$F_z = F\cos\theta_z$$

Forces in a Space

Forces on a Particle

Force Determination from its Line of Action



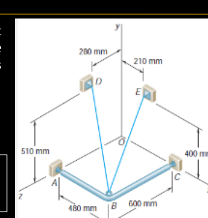
Steps:

$$\mathbf{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}$$
Unit Vector, $\boldsymbol{\lambda} = \mathbf{MN}/MN = (d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})/d$
$$\mathbf{F} = F\boldsymbol{\lambda}$$

If origin is not given, assume any suitable origin.

Problem 2.3 (Beer Johnston_10th edition_P2.89)

A frame *ABC is* supported in part by cable *DBE* that passes through a frictionless ring at *B.* Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D.*



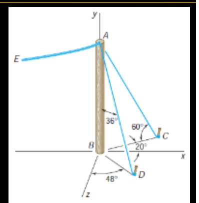
Solution:

DB = (480 mm) **i** – (510 mm) **j** + (320 mm) **k**
DB = 770 mm
Force on D, **F** = *F* **λ** = (240 N) **i** – (255 N) **j** + (160 N) **k**

Ans.: Components of force on D towards B exerted by the cable *DB*,
Fx = +240 N, *Fy* = -255 N, *Fz* = +160N

Problem 2.4 (Beer Johnston_10th edition_P2.77)

The end of the coaxial cable *AE is attached to the pole AB, which* is strengthened by the guy wires *AC and AD. Knowing that the* tension in wire *AC is 120 N, determine (a) the components of the* force exerted by this wire on the pole, (*b*) *the angles θx, θy, and θz* that the force forms with the coordinate axes.



Forces in a Space

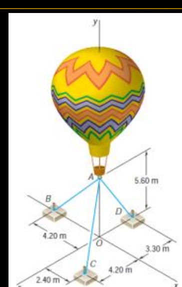
Forces on a Particle

Equilibrium

$$\sum \mathbf{F} = \sum F_x\mathbf{i} + \sum F_y\mathbf{j} + \sum F_z\mathbf{k} = \mathbf{0}$$
$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

Problem 2.5 (Beer Johnston_10th edition_P2.102)

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800 N vertical force at *A*, determine the tension in each cable.



Solution:

Draw the Free Body Diagram of point A.

Equilibrium Condition of A:

$\Sigma \mathbf{F} = \mathbf{0}$

So, $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = \mathbf{0}$

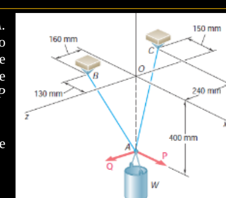
AB = -4.2 **i** - 5.6 **j**
AC = 2.4 **i** - 5.6 **j** + 4.2 **k**
AB = -5.6 **j** – 3.3 **k**

Ans.: T_{AB} = 201 N, T_{AC} = 372 N, T_{AD} = 416 N

Problem 2.6 (Beer Johnston_10th edition_P2.123)

A container of weight *W* is suspended from ring *A*. Cable *BAC* passes through the ring and is attached to fixed supports at *B* and *C*. Two forces *P* and *Q* are applied to the ring to maintain the container in the position shown. Knowing that *W* = 376 N, determine *P* and *Q*.

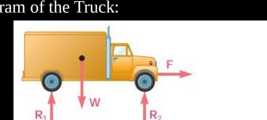
(Hint: The tension is the same in both portions of cable BAC.)



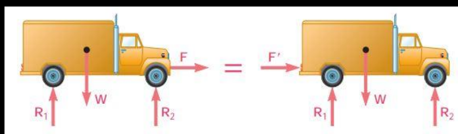
Statics of Rigid Bodies

Free Body Diagram

- Free Body Diagram of the Truck:



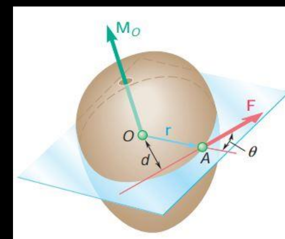
Principle of Transmissibility



Rigid Body Assumption

- **Deformation of the body under a force is negligible and has no effect on the condition of equilibrium or motion of the body.**

Moment of a Force About a Point



$$\mathbf{M_o} = \mathbf{r} \times \mathbf{F} = \mathbf{r_{A/O}} \times \mathbf{F}$$

$$M_o = rF \sin\theta = Fd$$

Problem 2.7 (Beer Johnston_10th edition_P3.25)

A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about A.

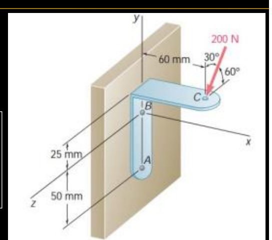
Solution:

$\mathbf{M_A} = \mathbf{r_{C/A}} \times \mathbf{F}$

$\mathbf{r_{C/A}}$ = (0.06 m) i + (0.075 m) j

F = - 200 N (cos 30°) j + 200 N (cos 60°) k

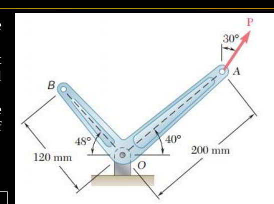
$\mathbf{M_A}$ = (7.5 Nm) i – (6 Nm) j – (10.39 Nm) k



Ans.: $\mathbf{M_A}$ = (7.5 Nm) i – (6 Nm) j – (10.39 Nm) k

Problem 2.8 (Beer Johnston_10th edition_P3.6)

A 300-N force **P** is applied at point *A* of the bell crank shown.
(a) Compute the moment of the force **P** about *O* by resolving it into horizontal and vertical components.
(b) Using the result of part a, determine the perpendicular distance from *O* to the line of action of *P*.



Solution:

(a)

$\mathbf{M_O} = \mathbf{r_{A/O}} \times \mathbf{F}$

$\mathbf{r_{A/O}}$ = (0.153209 m) i + (0.128558 m) j

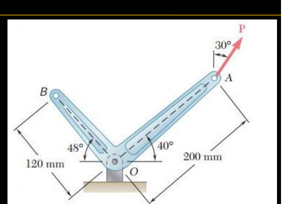
F = 300 N (sin 30°) i + 300 N (cos 30°) j

$\mathbf{M_O}$ = (20.521 Nm) k

Ans.: (a) $\mathbf{M_O}$ = (20.521 Nm) k or $\mathbf{M_O}$ = 20.521 Nm
(b) *d* = 68.4 mm

Problem 2.9 (Beer Johnston_10th edition_P3.7)

A 300-N force **P** is applied at point *A* of the bell crank shown.
Compute the moment of the force **P** about *O* by resolving it into along line *OA* and perpendicular to *OA*.



Moment of a Force About a Given Axis

- A scalar Quantity
- The projection of Moment on a given axis.

Moment of force about **Point *O***

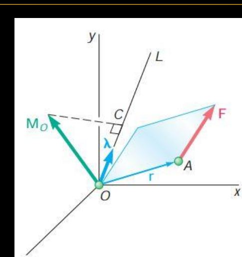
$\mathbf{M_O} = \mathbf{r_{A/O}} \times \mathbf{F}$

Moment of force about **Axis *OL***

$M_{OC} = \boldsymbol{\lambda} \cdot \mathbf{M_O}$

$M_{OC} = \boldsymbol{\lambda} \cdot (\mathbf{r_{A/O}} \times \mathbf{F})$

$$M_{OC} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



λ is the unit vector along *OL*

Problem 2.10 (Beer Johnston_10th edition_P3.59)

The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

Solution:

Moment of force $\mathbf{T}_{BH}$ about **Point *A***

$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BH}$

Moment of force $\mathbf{T}_{BH}$ about **Axis *AD***

$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \mathbf{M}_A$

$\mathbf{AD} = \mathbf{i} - 0.75\mathbf{j}$

$AD = 1.25$

$\boldsymbol{\lambda}_{AD} = (0.8\mathbf{i} - 0.6\mathbf{k})$

$\mathbf{r}_{B/A} = 0.5\mathbf{i}$

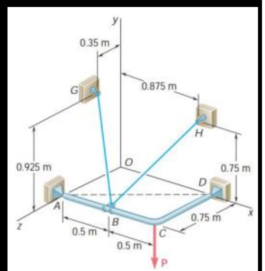
$\mathbf{BH} = 0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}$

$BH = 1.125$ m

$\boldsymbol{\lambda}_{BH} = (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})/3$

$\mathbf{T}_{BH} = 150\,(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$

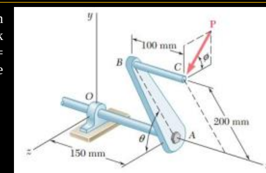
$M_{AD} = -90$ Nm



Ans.: -90 Nm

Problem 2.11 (Beer Johnston_10th edition_P3.53)

A single force ***P*** acts at *C* in a direction perpendicular to the handle *BC* of the crank shown. Knowing that M_x = +20 N m and M_y = -8.75 N m, and M_z = -30 N m. Determine the magnitude of ***P*** and the values of θ and ϕ.

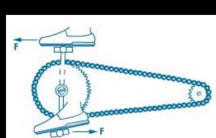


Ans.: **P** = 125 N, θ =53.1 °, ϕ = 73.7°

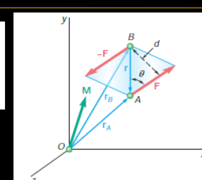
Force, Torque and Couple

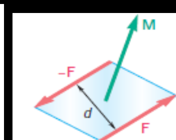
Applying Torque

Couple



Moment of a Couple





- Moment of a Couple = **Sum** of the moment of corresponding two forces about **ANY** point

$$\mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

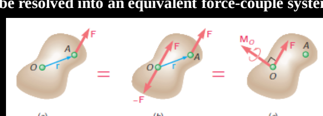
Value of the Moment, $M = rF \sin\theta = rd$

[*d* is the perpendicular distance between the line of action of the two forces]

Direction of the moment is perpendicular to the plane containing the two forces

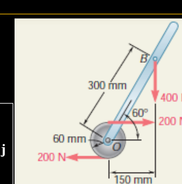
Force-Couple System

- **A force can be resolved into an equivalent force-couple system at any point.**



Problem 2.12 (Beer Johnston_10th edition_Sample Problem 3.7)

Replace the couple and force shown by an equivalent single force applied to the lever. Determine the distance from the shaft to the point of application of this equivalent force.



Solution:

Moment of F = 400 N force about **Point *O***

$\mathbf{M}_O = \mathbf{OB} \times \mathbf{F}$

$= [300\text{m} (\cos 60°)\,\mathbf{i} + 300\text{m} (\sin 60°)\,\mathbf{j}] \times (-400 \text{ N})\,\mathbf{j}$

$= (-60 \text{ Nm})\,\mathbf{k}$

Total Moment at *O*,

$\mathbf{M} = [-60 + (-200 \text{ N} * 120 \text{ mm})]\,\mathbf{k}$

$= (-84 \text{ Nm})\,\mathbf{k}$

So,

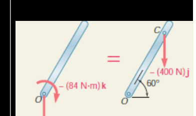
$\mathbf{OC} \times \mathbf{F} = \mathbf{M}$

$[(OC) \cos 60°\,\mathbf{i} + (OC) \sin 60°\,\mathbf{j}] \times (-400 \text{ N})\,\mathbf{j} = (-84 \text{ Nm})\,\mathbf{k}$

$[(OC) \cos 60° * (-400 \text{ N})]\,\mathbf{k} = (-84 \text{ Nm})\,\mathbf{k}$

$(OC) \cos 60° = 0.210 \text{ m} = 210 \text{ mm}$

$OC = 420$ mm Ans.: $OC = 420$ mm



End of Portion 2

References

- **Vector Mechanics for Engineers: Statics and Dynamics**
Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.